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# Estimation of pore diameter for intraparticle fluid flow in bidisperse porous chromatographic particles

G.A. Heeter, A.I. Liapis\*

Department of Chemical Engineering and Biochemical Processing Institute, University of Missouri-Rolla, Rolla, MO 65401-0249, USA

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## Abstract

A quantitative method is developed and presented for estimating the mean pore diameter,  $d_{\text{pore}}$ , that could allow intraparticle fluid flow in the macroporous region of adsorbent particles with a bidisperse porous structure packed in a column. The method is used to estimate the value of  $d_{\text{pore}}$  that is required to obtain specified values of the fraction,  $F$ , of the column superficial fluid velocity that is flowing through the macroporous region of the adsorbent particles and the value of  $d_{\text{pore}}$  that is required to obtain specified values of the intraparticle Peclet number,  $Pe_{\text{intra}}$ .

**Keywords:** Mathematical models; Adsorbents; Porous adsorbent particles; Intraparticle fluid flow; Perfusion chromatography; Pore size; Proteins

## 1. Introduction

As in previous publications [1–10], we define “perfusion chromatography” to refer to any chromatographic system in which the intraparticle velocity,  $v_p$ , is non-zero. Experimental evidence of intraparticle fluid flow in certain porous chromatographic particles has been recently presented [11]. Liapis et al. [5], Heeter and Liapis [7,9,10] and Xu and Liapis [8] have constructed and presented mathematical models that can be used to describe single-component liquid adsorption and desorption, as well as multi-component liquid adsorption in chromatographic columns packed with spherical perfusive or spherical purely diffusive adsorbent particles having a bidisperse porous structure. The model expressions [5,7–10] for the bidisperse porous adsorbent particles include the intraparticle mass transfer mecha-

nisms of convection and diffusion in the macroporous region and diffusion in the microporous region, as well as the mass transfer step involving the interaction between the adsorbate molecules and the active sites on the surface of the macropores and micropores. It should be noted here that the models of Liapis et al. [5], Heeter and Liapis [7,9,10] and Xu and Liapis [8] are not limited by the significant restrictions of other models of perfusion chromatography found in the literature [12–16], which assume infinitely fast interaction between adsorbate molecules and surface active sites and are valid only when the adsorption isotherm is linear and the feed to the chromatographic column can be represented by a Dirac pulse. Real chromatographic systems may not meet these restrictions.

In this work, a quantitative method is presented that can be used to estimate the value of the mean pore diameter that could allow intraparticle fluid flow ( $v_p > 0$ ) in the macroporous region of bidisperse

\*Corresponding author.

[1–10] porous adsorbent particles packed in a column.

## 2. Mathematical formulation

The spherical adsorbent particles with a bidisperse [4,5,7–10] porous structure are considered to have a microporous [4,5,7–10] region made by spherical microparticles (microspheres [4,5,7–10,12]) that are taken to be purely diffusive and a macroporous [4,5,7–10] region made by the macropores [1,3–5,7–10] in which intraparticle convection and pore diffusion occur. The expressions for the components of the intraparticle velocity,  $v_p$ , vector along the R and  $\theta$  directions ( $v_{pR}$  and  $v_{p\theta}$ , respectively) are as follows:

$$v_{pR} = FV_f \cos\theta \quad (1)$$

$$v_{p\theta} = -FV_f \sin\theta \quad (2)$$

Eqs. (1,2) are obtained [5,7,10] by using the expression of Neale et al. [17] for the stream function inside a permeable sphere in a packed bed and setting the parameter  $H$  of Neale et al. [17] equal to zero. Numerous parametric calculations [5,7,8,18,19] have indicated that for various adsorption systems, the value of  $H$  is essentially equal to zero. When  $v_{pR}$  and  $v_{p\theta}$  are given by Eqs. (1,2), respectively, the axial component of the intraparticle velocity,  $v_{px_1}$ , which is parallel to the flowing fluid stream along the axis of the column (see Fig. 1 in reference [5]) is as follows [5,7,10]:

$$v_{px_1} = FV_f \quad (3)$$

In Eq. (3), the parameter  $F$  can be considered to represent the fraction of the superficial fluid velocity in the column,  $V_f$ , that is flowing through the macroporous region of the adsorbent particle. The parameter  $F$  can be calculated from Eqs. (5), (7)–(13) in Ref. [10], which show that the parameter  $F$  depends upon the column void fraction,  $\varepsilon$ , and four structural parameters of the particle: the particle diameter,  $d_p$ , the macropore void fraction,  $\varepsilon_p$ , the microparticle diameter,  $d_m$  and the parameter  $\phi$ , which affects the value of the permeability of the adsorbent particle.

Common methods of measuring the pore size distribution of porous media, such as mercury porosimetry and adsorption methods, measure a “pore radius” that is directly proportional to the hydraulic radius of the porous medium [20,21]. The hydraulic radius,  $R_h$ , of a porous medium composed of uniform spheres is given by [22]

$$R_h = \frac{1}{6} \left( \frac{\varepsilon_m}{1 - \varepsilon_m} \right) d_s \quad (4)$$

where  $\varepsilon_m$  is the void fraction of the porous medium and  $d_s$  is the diameter of the spheres. The constant in the right-hand side of Eq. (4) could be different than  $1/6$  if the particles comprising the porous medium are not spherical in shape or are not uniform in size. With this in mind, the following relationship between the mean microparticle diameter,  $d_m$ , and the mean pore diameter,  $d_{pore}$ , of the macroporous region of bidisperse porous adsorbent particles is proposed:

$$d_m = \alpha \left( \frac{1 - \varepsilon_p}{\varepsilon_p} \right) d_{pore} \quad (5)$$

The value of the parameter  $\alpha$  could depend upon the shape of the microparticles and the distribution of sizes of the microparticles and includes the proportionality between  $d_{pore}$  and the hydraulic radius,  $R_h$ . A value of  $\alpha = 1.5$  has been reported [23] for uniform spheres in various packing arrangements.

For given values of  $\varepsilon$ ,  $\varepsilon_p$ ,  $d_p$ ,  $\phi$ ,  $F$  and  $\alpha$ , the value of  $d_{pore}$  can be estimated from Eq. (5) and Eqs. (5) and (7)–(13) in Ref. [10]. The value of  $d_{pore}$  obtained in this manner would represent an estimate of the mean pore diameter of the macroporous region of a perfusive adsorbent particle that is required to obtain the given value of the parameter  $F$ .

The intraparticle Peclet number,  $Pe_{intra}$ , is given by [5,7,10]

$$Pe_{intra} = \frac{FV_f d_p}{D_p} \quad (6)$$

where  $D_p$  represents the effective pore diffusion coefficient of the adsorbate in the pore fluid of the macroporous region. The value of  $F$  can be determined from Eq. (6) for given values of  $d_p$ ,  $D_p$ ,  $V_f$  and  $Pe_{intra}$ . Then, this value of  $F$  together with the given values of the parameters  $\varepsilon$ ,  $\varepsilon_p$ ,  $d_p$ ,  $\phi$  and  $\alpha$  could be used to estimate the value of the mean pore

diameter,  $d_{\text{pore}}$ , from Eq. (5) and equations 5 and 7–13 in reference [10]. The value of  $d_{\text{pore}}$  obtained in this manner would represent an estimate of the mean pore diameter of the macroporous region of a perfusive adsorbent particle that is required to obtain the given value of  $Pe_{\text{intra}}$ .

### 3. Results and discussion

In Fig. 1, the value of the mean pore diameter,  $d_{\text{pore}}$ , of the macroporous region required to obtain a given value of the parameter  $F$  is presented, for different values of the porosity,  $\varepsilon_p$ , of the macroporous region and for different values of the parameter  $\alpha$  in Eq. (5), when  $\varepsilon = 0.35$ ,  $d_p = 15 \cdot 10^{-6}$  m and  $\phi = 150$ . It can be clearly observed that, for each set of values of  $\varepsilon_p$  and  $\alpha$ , the value of  $d_{\text{pore}}$  required to obtain a given value of  $F$  increases as  $F$  is increased, which can be interpreted as simply that larger pores are needed to obtain more intraparticle fluid flow. The value of  $d_{\text{pore}}$  required to obtain a given value of  $F$  increases as  $\varepsilon_p$  is decreased, which means that, in order to have the same magnitude of intraparticle

fluid flow when  $\varepsilon_p$  is decreased, larger pores are required. Furthermore, for each value of  $\varepsilon_p$ , the value of  $d_{\text{pore}}$  required to obtain a given value of  $F$  is higher for  $\alpha = 0.25$  than for  $\alpha = 1.5$ , because the microsphere diameter,  $d_m$ , is the same for both cases. It should be emphasized here that the results in Fig. 1 were obtained by keeping the parameters  $\alpha$  and  $\phi$  constant when  $\varepsilon_p$  was changed and keeping  $\varepsilon_p$  and  $\phi$  constant when  $\alpha$  was changed. However, changes made in the process used to manufacture the porous adsorbent particles in order to create adsorbent particles with different values of  $\varepsilon_p$  could change (a) the shape of the microparticles, (b) the distribution in the sizes of the microparticles, (c) the packing arrangement of the microparticles and (d) the pore connectivity of the macroporous region. This could require the value of  $\alpha$  and/or the value of  $\phi$  to change along with the value of  $\varepsilon_p$ , which could change how the value of  $d_{\text{pore}}$  varies with changes in the value of  $\varepsilon_p$ .

In Fig. 2, the value of  $d_{\text{pore}}$  required to obtain  $Pe_{\text{intra}} = 1$  and  $Pe_{\text{intra}} = 10$  for a given value of the superficial fluid velocity,  $V_f$ , in the column is presented, for three different adsorbates [lysozyme,

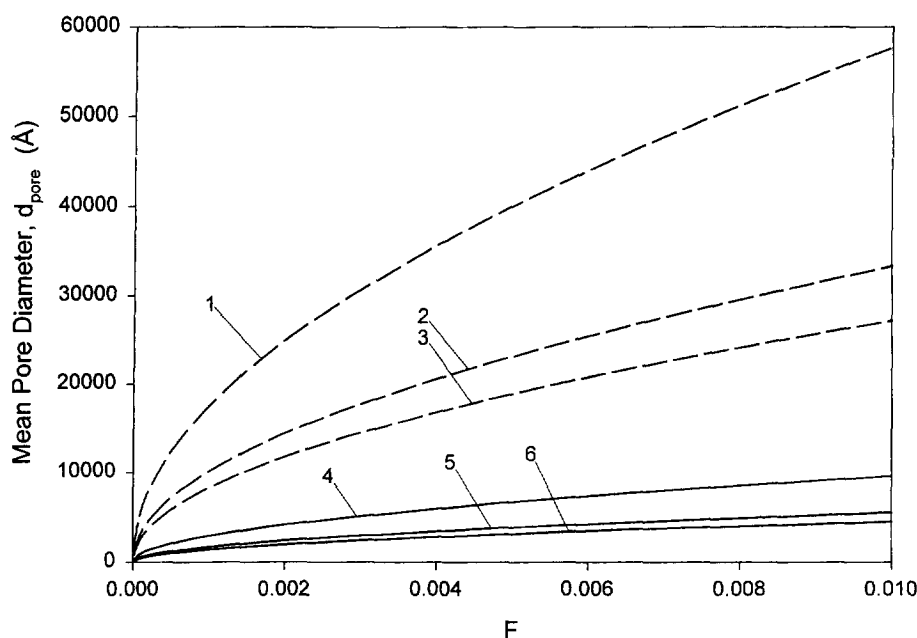


Fig. 1. Estimated mean pore diameter,  $d_{\text{pore}}$ , of the macroporous region required to obtain a given value of the parameter  $F$  for different values of  $\varepsilon_p$  and  $\alpha$  when  $\varepsilon = 0.35$ ,  $d_p = 15 \cdot 10^{-6}$  m and  $\phi = 150$ . The solid lines are for  $\alpha = 1.5$  and the dashed lines are for  $\alpha = 0.25$ . (1)  $\varepsilon_p = 0.10$ ,  $\alpha = 0.25$ ; (2)  $\varepsilon_p = 0.30$ ,  $\alpha = 0.25$ ; (3)  $\varepsilon_p = 0.45$ ,  $\alpha = 0.25$ ; (4)  $\varepsilon_p = 0.10$ ,  $\alpha = 1.5$ ; (5)  $\varepsilon_p = 0.30$ ,  $\alpha = 1.5$ ; (6)  $\varepsilon_p = 0.45$ ,  $\alpha = 1.5$ .

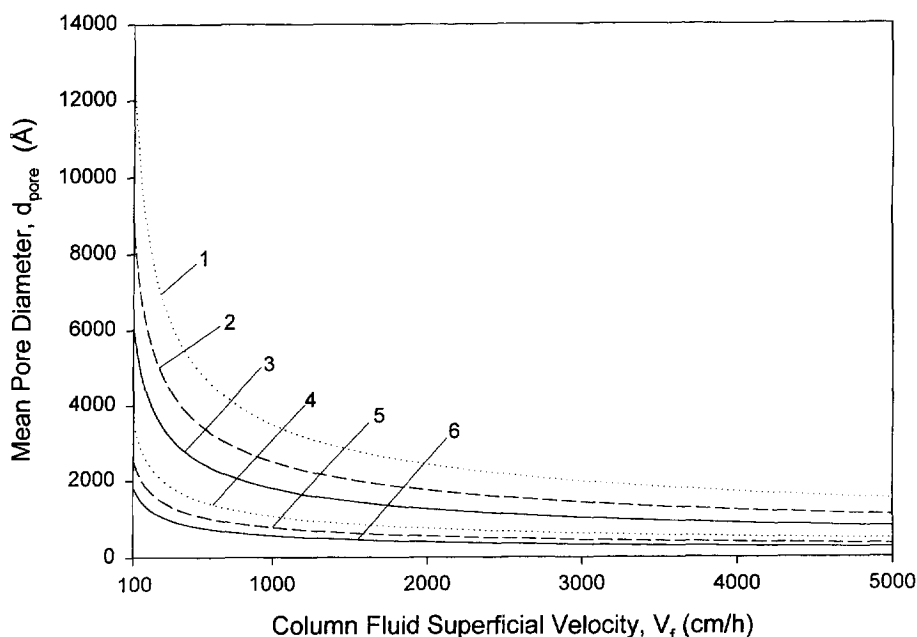


Fig. 2. Estimated mean pore diameter,  $d_{\text{pore}}$ , of the macroporous region required to obtain  $Pe_{\text{intra}} = 1$  and  $Pe_{\text{intra}} = 10$  for a given value of the column superficial velocity,  $V_f$ , for three different adsorbates when  $\varepsilon = 0.35$ ,  $d_p = 15 \cdot 10^{-6}$  m,  $\phi = 150$  and  $\alpha = 1.5$ . The solid curves are for  $\beta$ -galactosidase, the dashed curves are for BSA and the dotted curves are for lysozyme. (1)  $Pe_{\text{intra}} = 10$ , lysozyme; (2)  $Pe_{\text{intra}} = 10$ , BSA; (3)  $Pe_{\text{intra}} = 10$ ,  $\beta$ -galactosidase; (4)  $Pe_{\text{intra}} = 1$ , lysozyme; (5)  $Pe_{\text{intra}} = 1$ , BSA; (6)  $Pe_{\text{intra}} = 1$ ,  $\beta$ -galactosidase.

bovine serum albumin (BSA) and  $\beta$ -galactosidase], when  $\varepsilon = 0.35$ ,  $d_p = 15 \cdot 10^{-6}$  m,  $\varepsilon_p = 0.45$ ,  $\phi = 150$  and  $\alpha = 1.5$ . The results of earlier work [5,7] have shown that, when  $Pe_{\text{intra}} = 1$ , although intraparticle fluid flow exists, the magnitude of the intraparticle fluid flow is not large enough to affect the performance of the chromatographic system, whereas, when  $Pe_{\text{intra}} = 10$ , the effect of intraparticle fluid flow on the performance of the chromatographic system could be significant. The effective pore diffusion coefficient,  $D_p$ , for each adsorbate was estimated from the expression  $D_p = \varepsilon_p D_{\text{mf}} / \tau$ , where  $D_{\text{mf}}$  is the free molecular diffusivity of the adsorbate and  $\tau$  represents the tortuosity factor ( $\tau > 1$ ) of the pores of the macroporous region of the adsorbent particles; in this work, the value of  $\tau$  was taken to be, in all cases, equal to 2.0, and the values of the free molecular diffusivities,  $D_{\text{mf}}$  (evaluated at 20°C), of lysozyme, BSA and  $\beta$ -galactosidase were [24]  $11.2 \times 10^{-11}$  m<sup>2</sup>/s,  $5.90 \times 10^{-11}$  m<sup>2</sup>/s and  $3.12 \times 10^{-11}$  m<sup>2</sup>/s, respectively.

The results in Fig. 2 indicate that, for every value of  $V_f$ , the value of  $d_{\text{pore}}$  required to obtain a given value of  $Pe_{\text{intra}}$  is lower for the system with  $\beta$ -galactosidase than the value of  $d_{\text{pore}}$  for the system with BSA, which, in turn, is lower than the value of  $d_{\text{pore}}$  for the system with lysozyme. This happens because  $\beta$ -galactosidase has the smallest value of the effective pore diffusion coefficient,  $D_p$ , and this results in the smallest value of  $F$  from Eq. (6), when the same values for  $Pe_{\text{intra}}$ ,  $V_f$  and  $d_p$  are used for the three different adsorbates. Furthermore, for the same values of  $\varepsilon$ ,  $\varepsilon_p$ ,  $d_p$ ,  $\phi$  and  $\alpha$ , the smallest value of  $d_{\text{pore}}$  is obtained when the value of  $F$  is smallest, as the results in Fig. 1 indicate. Therefore, the adsorbate with the smallest value of  $D_p$  ( $\beta$ -galactosidase) requires the smallest value of  $d_{\text{pore}}$  to obtain a given value of  $Pe_{\text{intra}}$ , because it has the smallest value of  $F$ , and the adsorbate with the largest value of  $D_p$  (lysozyme) requires the largest value of  $d_{\text{pore}}$  to obtain a given value of  $Pe_{\text{intra}}$ , because it has the largest value of  $F$ .

Additional results are presented in reference [25].

#### 4. Conclusion

A method for estimating the mean pore diameter,  $d_{\text{pore}}$ , that could allow intraparticle fluid flow in the macroporous region of bidisperse porous adsorbent particles packed in a column, was presented. This method can be used to evaluate the potential importance of intraparticle convection in an adsorption system with minimal time and effort.

#### 5. Symbols

|                     |  |
|---------------------|--|
| $d_m$               | diameter of spherical microparticle, m   |
| $d_p$               | diameter of spherical porous adsorbent particle, m   |
| $d_{\text{pore}}$   | mean pore diameter of macroporous region, m  |
| $d_s$               | diameter of spherical particles in porous medium in Eq. (4), m                             |
| $D_{\text{mf}}$     | free molecular diffusivity of adsorbate, $\text{m}^2/\text{s}$                             |
| $D_p$               | effective pore diffusion coefficient of adsorbate in the macropores, $\text{m}^2/\text{s}$ |
| $F$                 | parameter given by equation 5 in reference [10], dimensionless                             |
| $H$                 | parameter given by equation 6 in reference [10]  |
| $Pe_{\text{intra}}$ | intraparticle Peclet number, dimensionless   |
| $R$                 | radial distance in adsorbent particle, m   |
| $R_h$               | hydraulic radius of porous medium in Eq. (4), m  |
| $v_p$               | intraparticle velocity vector, m/s   |
| $v_{pR}$            | intraparticle velocity component along the R direction, m/s                                |
| $v_{p\theta}$       | intraparticle velocity component along the $\theta$ direction, m/s                         |
| $v_{px_1}$          | axial component of the intraparticle velocity, m/s   |
| $V_f$               | column fluid superficial velocity, m/s   |

##### 5.1. Greek letters

|                 |  |
|-----------------|--|
| $\alpha$        | parameter in Eq. (5)                                   |
| $\varepsilon$   | void fraction in column                                |
| $\varepsilon_m$ | void fraction of porous medium in Eq. (4)              |
| $\varepsilon_p$ | macropore void fraction                                |
| $\theta$        | polar coordinate angle, radians                        |
| $\tau$          | tortuosity factor of macroporous region, dimensionless |
| $\phi$          | parameter in Eq. (13) in Ref. [10]                     |

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